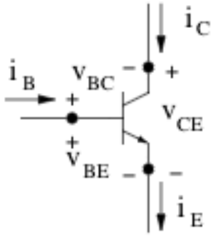
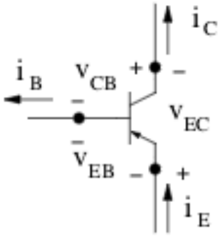
BJT Large and Small Signal Models

- This document contains large and small signal models for the BJT transistor in the active region.
- I made sure to include models that model the Early Effect.

Summary of BJT Large-Signal Models:

	NPN	PNP
Cut-off:		
BE reverse biased	$i_B = 0,$	$i_B = 0$
	$i_C = 0$	$i_C = 0$
Active Linear:		
BE forward biased	$i_B = \frac{I_S}{\beta}\, e^{v_{BE}/nV_T}$	$i_B = \frac{I_S}{\beta}\, e^{v_{EB}/nV_T}$
CE reverse biased	$i_C = I_S\, e^{v_{BE}/nV_T}\left(1 + \frac{v_{CE}}{V_A}\right)$	$i_C = I_S\, e^{v_{EB}/nV_T}\left(1 + \frac{v_{EC}}{V_A}\right)$
Saturation:		
BE forward biased	$i_B = \frac{I_S}{\beta}\, e^{v_{BE}/nV_T}$	$i_B = \frac{I_S}{\beta}\, e^{v_{EB}/nV_T}$
CE forward biased	$i_C < \beta i_B$, $v_{CE} = 0.1 - 0.3$ V	$i_C < \beta i_B$, $v_{EC} = 0.1 - 0.3$ V





$$i_C + i_B = i_E$$

$$i_C = \beta i_B \left(1 + \frac{v_{CE}}{V_A}\right)$$

5.8.7 MODELING THE EARLY EFFECT

The dependence of the collector current on collector-emitter voltage is easily included in the simplified mathematical model for the forward-active region of the BJT by modifying Eqs. (5.23) as follows:

$$
\begin{aligned}
i_C &= I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]\left[1+\frac{v_{CE}}{V_A}\right] \\
\beta_F &= \beta_{FO}\left[1+\frac{v_{CE}}{V_A}\right] \qquad (5.43) \\
i_B &= \frac{I_S}{\beta_{FO}}\left[\exp\left(\frac{v_{BE}}{V_T}\right)\right]
\end{aligned}
$$

β_{FO} represents the value of β_F extrapolated to $V_{CE} = 0$. In these expressions, the collector current and current gain now have the same dependence on v_{CE}, but the base current remains independent of v_{CE}. This is consistent with Fig. 5.28, in which the separation of the constant-base-current curves in the forward-active region increases as v_{CE} increases, indicating that the current gain β_F is increasing with v_{CE}.

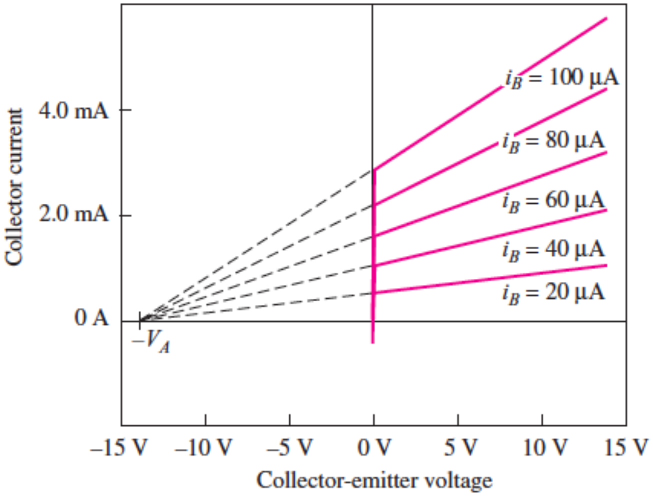
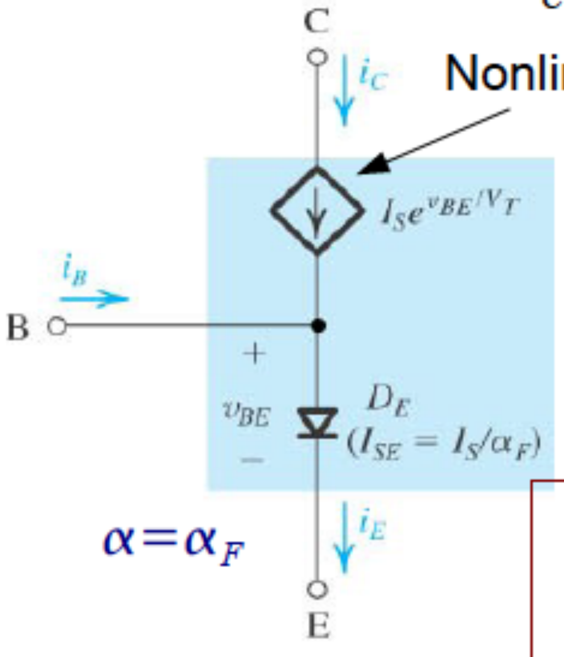


Figure 5.28 Transistor output characteristics identifying the Early voltage V_A.

Ignoring Early Voltage

Two equivalent large signal circuit models for the ***forward-active mode*** *NPN BJT:*

$$i_C = I_S\left(e^{\frac{v_{BE}}{V_T}} - 1\right) \approx I_S e^{\frac{v_{BE}}{V_T}}$$



Nonlinear VCCS

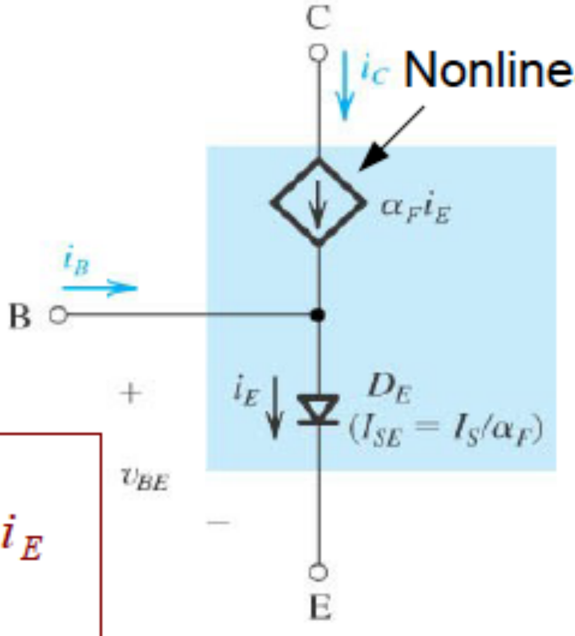
$\alpha = \alpha_F$

(a)

Key Eqs.

$$i_C \approx I_S e^{\frac{v_{BE}}{V_T}} = \alpha_F i_E$$

$$i_E \approx \frac{I_S}{\alpha_F} e^{\frac{v_{BE}}{V_T}}$$



Nonlinear CCCS

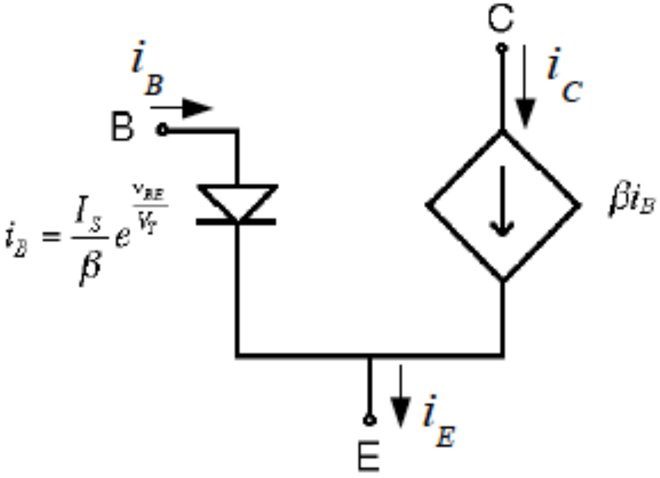
(b)

Ignoring Early Voltage

Yet another NPN BJT large signal model

$$i_C = \beta i_B \approx I_S e^{\frac{v_{BE}}{V_T}} \Rightarrow i_B \approx \frac{I_S}{\beta} e^{\frac{v_{BE}}{V_T}}$$

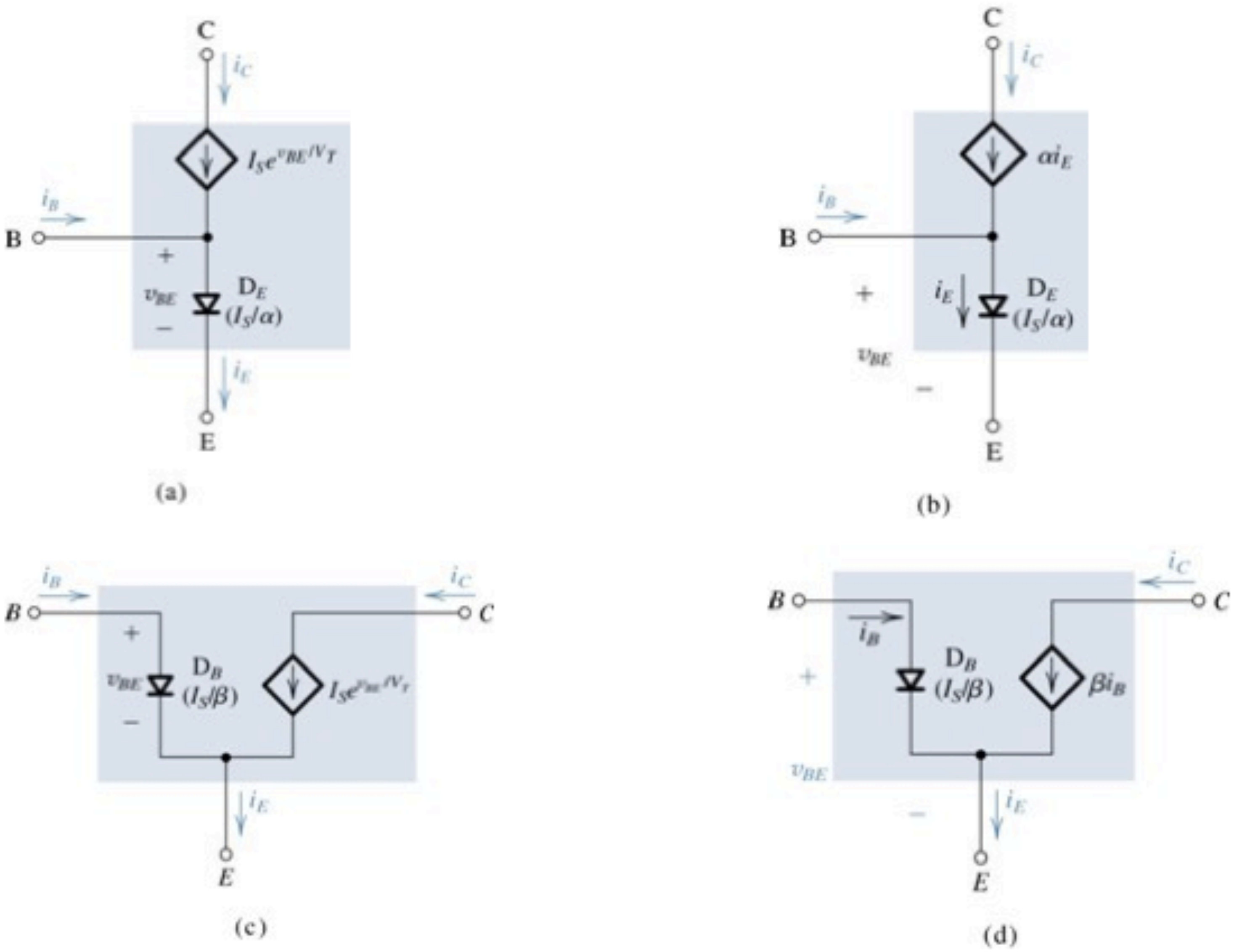
This "looks like" a diode between base and emitter and the equivalent circuit becomes:



Note that in this model, the diode current is represented in terms of the *base current*. In the previous ones, it was represented in terms of the *emitter current*.

Equivalent Circuit Models

Ignoring Early Voltage



Large-signal equivalent-circuit models of the *npn* BJT operating in the active mode.

Ignoring Early Voltage

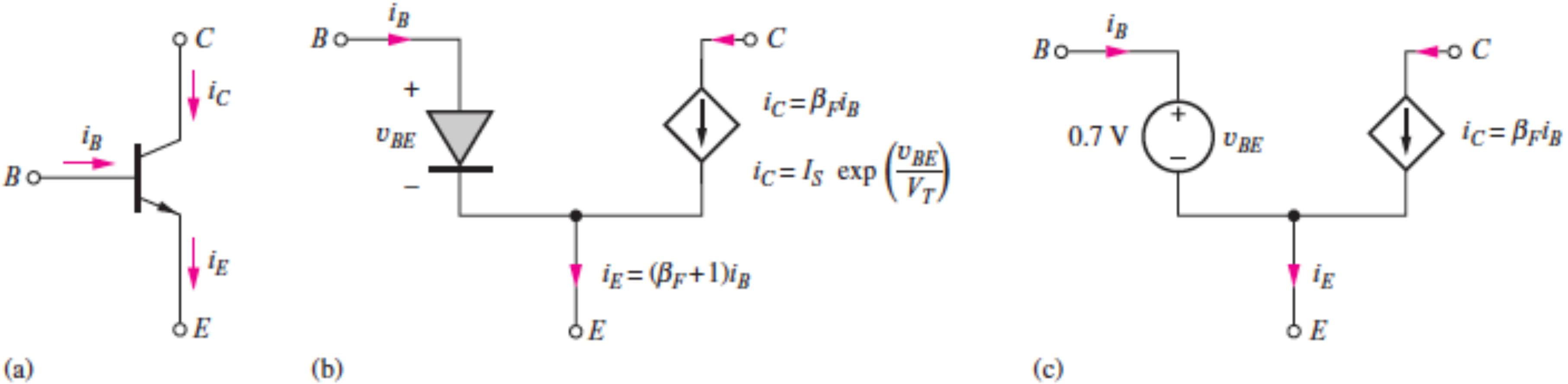
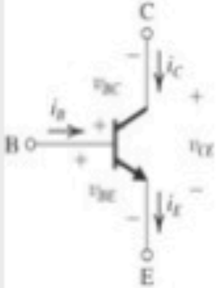
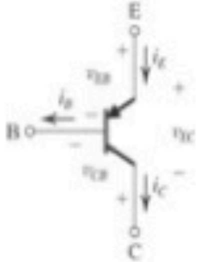
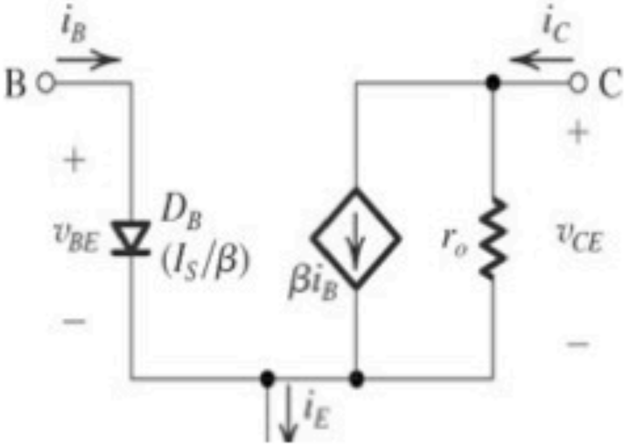


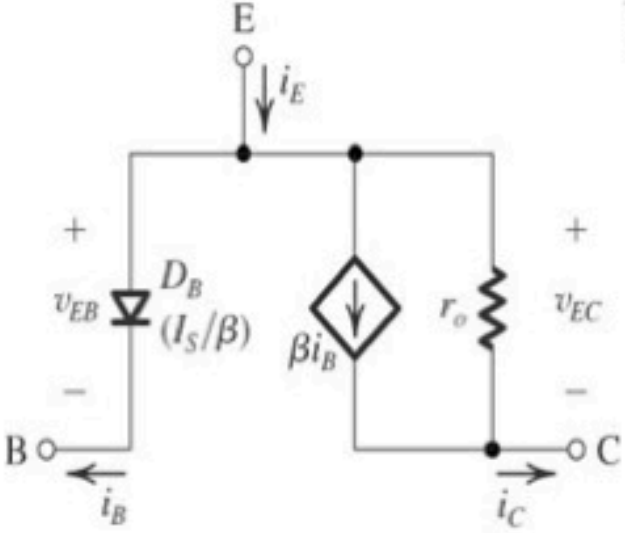
Figure 5.17 (a) *npn* transistor. (b) Simplified model for the forward-active region. (c) Further simplification for the forward-active region using the CVD model for the diode.

Table 5.3 Symbols & Large Signal Model









Large-Signal Equivalent-Circuit Model (Including the Early Effect)

$$i_B = \left(\frac{I_S}{\beta}\right)e^{v_{BE}/V_T}$$

$$i_B = \left(\frac{I_S}{\beta}\right)e^{v_{EB}/V_T}$$

$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

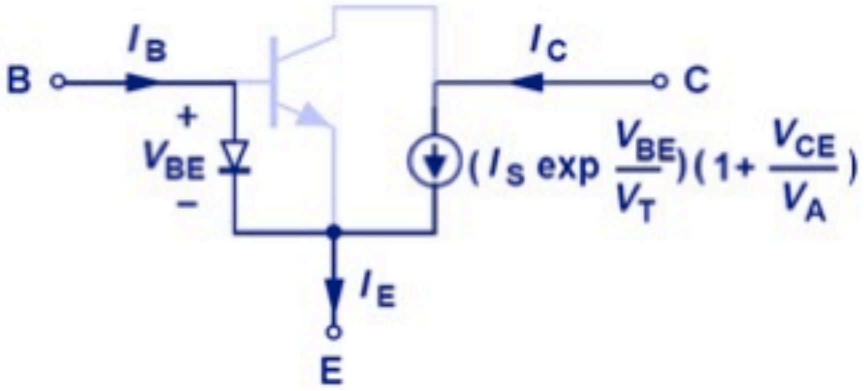
$$i_C = I_S e^{v_{EB}/V_T}\left(1 + \frac{v_{EC}}{|V_A|}\right)$$

$$r_o = V_A/(I_S e^{V_{BE}/V_T})$$

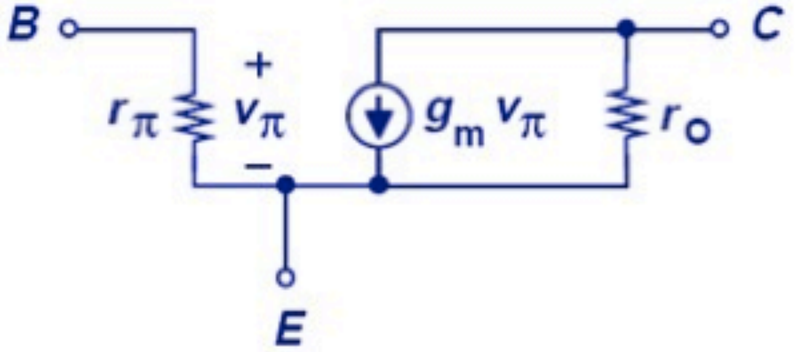
$$r_o = |V_A|/(I_S e^{V_{EB}/V_T})$$

Large-Signal *vs.* Small-Signal Models

- The large-signal model is used to determine the DC operating point (V_{BE}, V_{CE}, I_B, I_C) of the BJT.

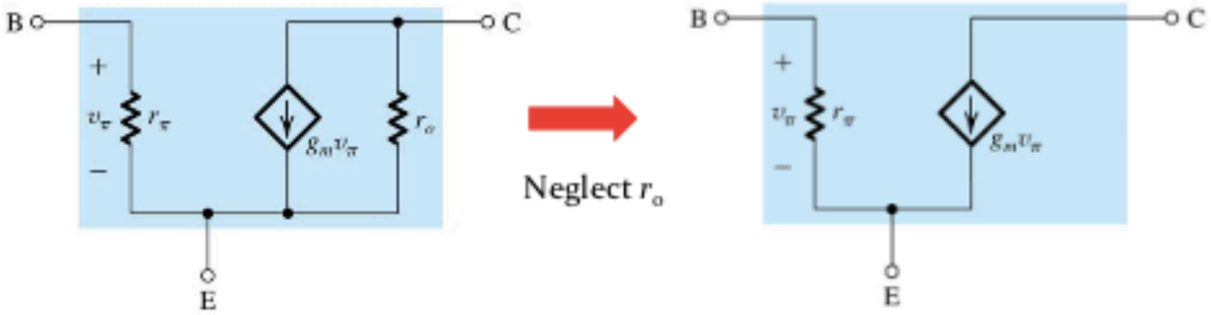


- The small-signal model is used to determine how the output responds to an input signal.



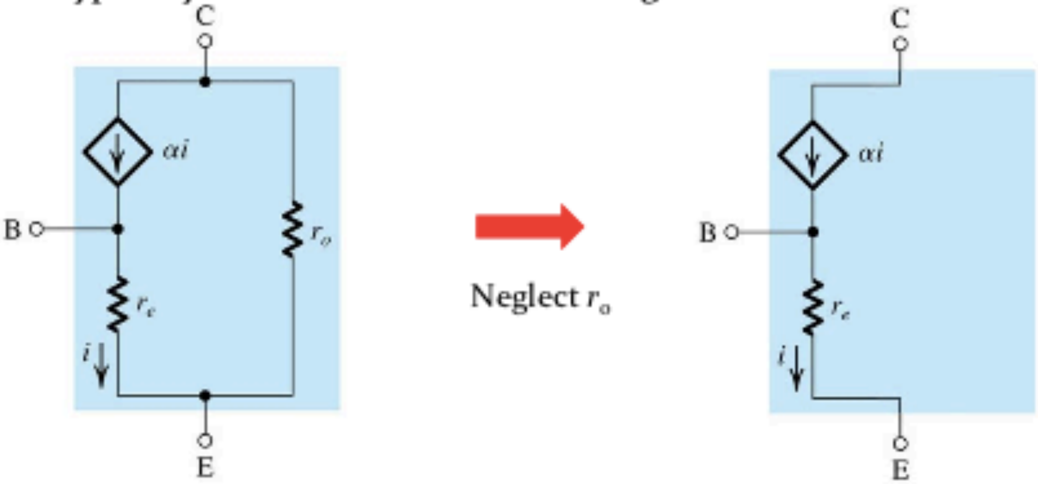
BJT small-signal models

- ❑ Two models are exchangeable and does not affect the analysis result
- ❑ The hybrid-π model
  - ■ Typically used as the emitter is grounded



Neglect r_o

- ❑ The T model
  - ■ Typically used as the emitter is not grounded



Neglect r_o

Model Parameters in Terms of DC Bias Currents

$$g_m = \frac{I_C}{V_T} \qquad r_e = \frac{V_T}{I_E} = \alpha\frac{V_T}{I_C}$$

$$r_\pi = \frac{V_T}{I_B} = \beta\frac{V_T}{I_C} \qquad r_o = \frac{|V_A|}{I_C}$$

In Terms of g_m

$$r_e = \frac{\alpha}{g_m} \qquad r_\pi = \frac{\beta}{g_m}$$

In Terms of r_e

$$g_m = \frac{\alpha}{r_e} \qquad r_\pi = (\beta+1)r_e$$

Relationships between α and β

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1}$$

Output Resistance

r_o is output resistance seen from collector terminal

$$i_C = I_S e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right)$$

$$r_o = \left[\left.\frac{\partial i_C}{\partial V_{CE}}\right|_{V_{BE}=constant}\right]^{-1}$$

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

Alternatively, neglecting the Early effect on the collector current, we define

$$I_C' = I_S e^{V_{BE}/V_T}$$

The output resistance then becomes

$$r_o = \frac{V_A}{I_C'}$$